Dielectric-Lined Circular Waveguide with Increased Usable Bandwidth

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Summary—This report describes how the useful (single-mode) bandwidth of circular waveguide can be significantly increased by using a smaller diameter with dielectric lining. The increased bandwidth results from an increase in the ratio of the cutoff frequency of the second, or TM-01, mode over that of the dominant, or TE-11 mode. A cavity test is described which permits the measurement of mode separation for any thickness of dielectric lining. A lining with a dielectric constant in the vicinity of 4 appears to be near optimum for maximum single-mode bandwidth. For a dielectric constant of 3.8 (fused silica), the greatest mode separation is obtained if the dielectric cross section has about 0.44 of the area of the entire cross section, in which case the useful bandwidth is about doubled (perhaps increased from 8 to 16 per cent, if the entire band is to clear the TE-11 cutoff frequency by 20 per cent). Over the useful bandwidth of such a waveguide, the guide wavelength may be calculated approximately by the simple waveguide formula in terms of the equivalent dielectric constant, as determined by the cavity measurement.

Applications which would benefit from increased dominant-mode bandwidth in circular waveguide include rotary joints carrying circular polarization, rotary attenuators and phase shifters, and array radiators.

I. Introduction

IN A CIRCULAR metal pipe used as an electromagnetic waveguide, there are discrete modes of propagation [1]. For each mode there is a critical (cutoff) frequency below which waves are not propagated. In practice, it is generally desirable to restrict the propagation to a single mode. If the dominant, TE-11, mode is to be utilized, the waveguide diameter may be chosen such that the next higher-order mode, the TM-01 mode, has its cutoff frequency above the operating frequency band. Thus the cutoff frequency of the TM-01 mode imposes an upper limit on the waveguide's useful bandwidth. The lower limit of this bandwidth is somewhat above the TE-11 cutoff frequency, since the propagation is impaired near cutoff. Nevertheless, the ratio of the TM-01 cutoff frequency over the TE-11 is an indication of the waveguide's useful bandwidth. This ratio is 1.31 for a uniform waveguide completely filled with any homogeneous isotropic dielectric. In such a waveguide, the third mode, the TE-21, has a cutoff frequency 1.66 times that of the dominant mode.

A study has been conducted with the objective of increasing the useful bandwidth of circular waveguide, by increasing the separation between the cutoff frequencies of the dominant mode and the next higher modes, as a result of dielectric lining.

The following sections will discuss the effect of dielectric lining, and describe an experimental procedure which has determined the filling fraction for greatest mode separation in a dielectric-lined circular waveguide.

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A justification of the basic assumptions and the experimental procedure is presented.

II. Effect of Partial Dielectric Filling

The cutoff frequency of a cylindrical waveguide is determined by its transverse geometry or cross section. Our object is to select a waveguide cross section which gives the desired mode separation.

Let us consider the electric-field configuration of the first three modes in circular waveguide. As shown in Fig. 1, the TM-01 differs from the TE-11 mode in that it has an axial E-field component, and this component is maximum near the axis of the waveguide. Therefore it might be expected that peripheral loading would reduce the cutoff frequency of the TM-01 mode less than that of the dominant mode, and this proves to be true. Peripheral loading would also reduce the TE-21 cutoff, even more than the dominant-mode cutoff. The TE-21 cutoff might not be reduced sufficiently to put it below the TM-01; hence it might not impose a limitation on the useful bandwidth.

Fig. 2 shows three examples of peripheral loading with cylindrical configurations of dielectric or metal. Other suitable configurations might include combinations of the three types depicted. The present study is restricted to the dielectric-lined waveguide as shown in Fig. 2(a).

The problem now reduces to confirming that greater mode separation is obtained in the dielectric-lined waveguide, and then finding the optimum filling fraction. Inhomogeneously filled waveguides have received sporadic attention in the literature. General considerations and theoretical methods of solution have been presented [4], [7], [10]. The problem of the dielectric-lined waveguide has been solved for the circularly symmetric modes [2], [6], [9]. Solutions have been given for other modes if the lining is thin [8], [11]. It is especially difficult to obtain a general solution for the dominant mode, which does not retain its pure TE character if the waveguide is filled inhomogeneously. Although the dominant mode in lined waveguide is a hybrid mode, it will here be referred to simply as the TE-11 mode, since this is the limit it approaches as the lining becomes very thin or very thick, or near cutoff in any case. One reference does treat the dominant mode in an inhomogeneously filled circular waveguide [5]. Unfortunately, the problem was solved only for axial dielectric loading, and not for the geometry under consideration here.

The mathematical difficulties in an exact theoretical solution of our problem led us to investigate experi-

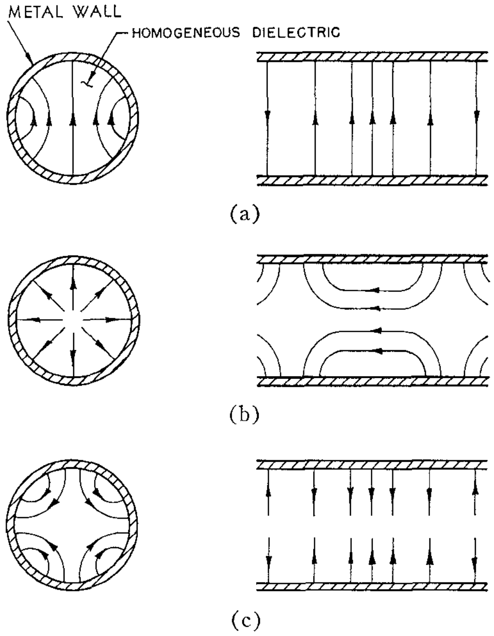


Fig. 1—Electric-field configuration for the first three modes in homogeneously filled circular waveguide. (a) Dominant mode: TE-11. (b) Second mode: TM-01. (c) Third mode: TE-21.

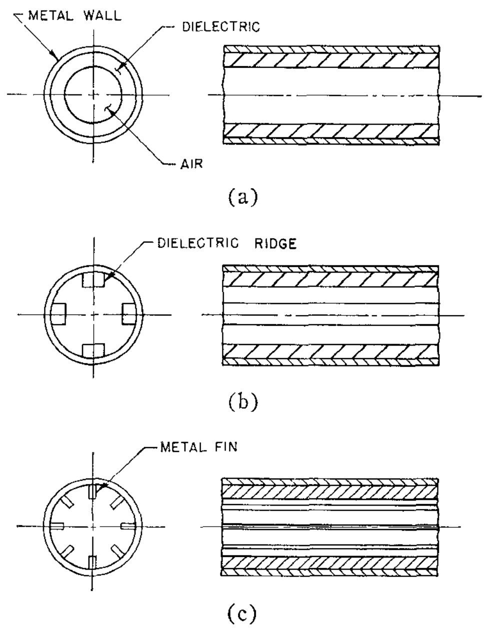


Fig. 2—Cylindrical configurations in circular waveguide for increasing the separation between the first two modes. (a) Dielectric lining. (b) Dielectric ridges. (c) Metal fins.

mental approaches. Let us hypothesize that a metal pipe lined with a material having a dielectric constant $k_d > 1$ may be regarded, for any one mode, as a metal pipe of the same diameter, homogeneously filled with a material having an "equivalent dielectric constant" k_e, where $k_d > k_e > 1$. The guide-wavelength is then

$$\lambda_g = \frac{\lambda}{\sqrt{k_e - (\lambda/\lambda_{0c})^2}} \tag{1}$$

where λ is the free-space wavelength and λ_{0c} is the cutoff wavelength of an empty metal pipe having the same diameter. The cutoff frequency is given by

$$f_c = \frac{v_0}{\lambda_{0c}\sqrt{k_e}} \tag{2}$$

where v_0 is the velocity of light in a vacuum. Solving (1) for k_e we obtain

$$k_e = \left(\frac{\lambda}{\lambda_{0c}}\right)^2 + \left(\frac{\lambda}{\lambda_g}\right)^2. \tag{3}$$

Substituting in (2) we have for reference further on

$$f_c = \frac{v_0}{\lambda\sqrt{1 + (\lambda_{0c}/\lambda_g)^2}}. \tag{4}$$

Let k_e for the dominant mode be k_{e1}, and k_e for the second mode be k_{e2}. The useful bandwidth will increase if $k_{e1} > k_{e2}$ and the third, or TE-21, mode still does not propagate. The mode separation may be measured by observing the resonances in a dielectric-lined cavity as discussed in Section III.

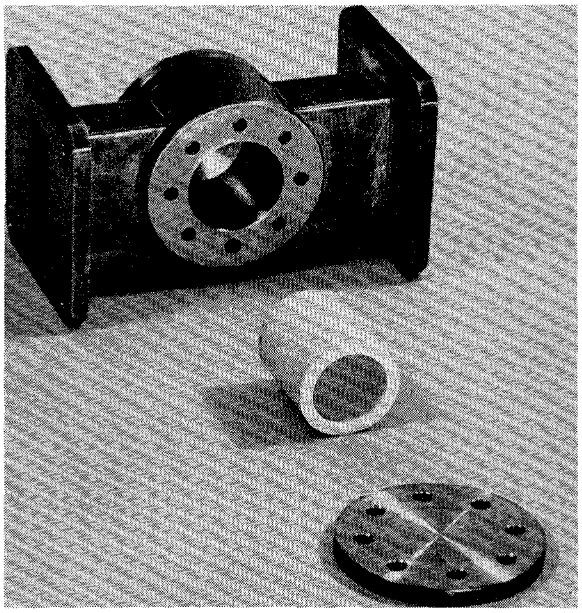
III. Experimental Determination of Mode Separation

A cavity resonator [3] will be formed if two separated conducting planes are passed through the lined waveguide, perpendicular to its axis. If a cavity of known dimensions is filled with a homogeneous dielectric, the dielectric constant of the filler may be computed from an observed resonant frequency. This procedure may be extended to an inhomogeneously filled cavity by applying the concept of "equivalent dielectric constant," which has been discussed in Section II. From the frequencies at which the cavity resonates, we can measure mode separation as a function of lining thickness.

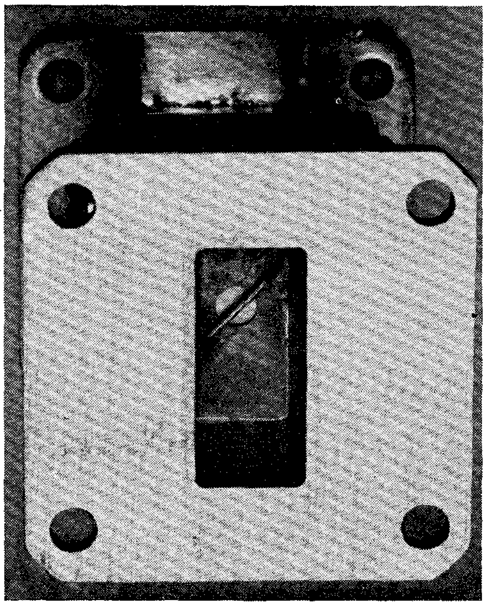
The frequency at which a cavity resonates may be determined by a transmission test: At resonance, there is maximum transmission from a generator, through the cavity, to a detector. Fig. 3(a) shows the cavity used to determine mode separation; one cover plate and the dielectric lining are shown removed. On either side of the cylindrical cavity are rectangular waveguides, one of which connects to a generator, and the other to a detector. An end view, Fig. 3(b), shows a coupling hole which connects the cavity to the ordinary rectangular waveguide. A wire positioned diagonally across the hole provides coupling with both TE and TM modes in the cavity.

To facilitate the observation of resonances, a sweeping oscillator and oscilloscope were used to obtain a plot of cavity transmission as a function of frequency. The cavity's resonant frequencies, as determined by a precision wavemeter, were measured for dielectric inserts of various materials and inner diameters.

The dimensions of the cavity were chosen such that



(a)



(b)

Fig. 3—Photographs of the test cavity. (a) Side view, dielectric lining removed. (b) End view, showing coupling hole.

the lowest resonance occurs in the dominant or TE-11 mode and the next higher resonance occurs in the TM-01 mode. The first resonance reduces to the TE-111 mode for a homogeneously filled cavity and occurs when the axial length of the cavity is one-half the guide wavelength in the dominant mode. With the aid of (4), we may compute the dominant mode's cutoff frequency. (The validity of this equation will be discussed in Section IV.) The second cavity resonance is the TM-010, which occurs at the TM-01 mode's cutoff frequency.

The ratio of the TM-01 cutoff frequency over that of the TE-11 mode was determined, as outlined above, for various dielectric materials and fractions of filling. In

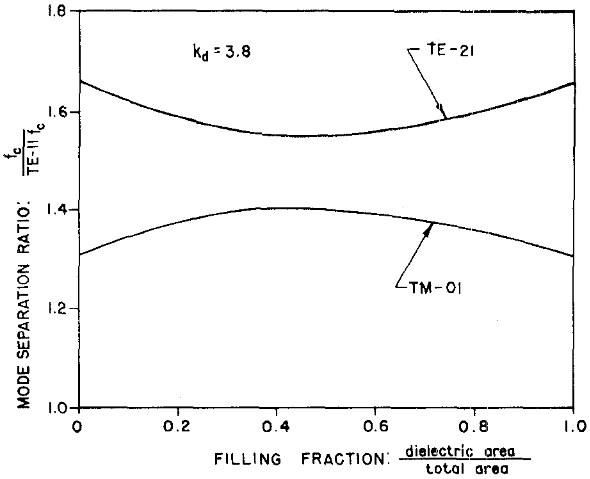


Fig. 4—Separation between dominant and higher-order modes vs filling fraction.

certain cases, additional observations were made to determine the separation between the TE-11 and TE-21 cutoff frequencies. Fig. 4 shows how the separation between the dominant and the next higher modes varies with filling fraction, that is, the area ratio of the dielectric cross section over the entire circular cross section. As the filling fraction increases from zero, the relative spacing between the TM-01 and TE-11 cutoffs increases, while the relative spacing between the TE-21 and TE-11 decreases. The curves level off as the filling fraction approaches 0.5, and of course return to their original positions as the filling fraction approaches 1.0, another case of homogeneous filling. With $k_d = 3.8$, the TE-21 cutoff frequency in no instance is less than that of the TM-01, thus the TE-21 cutoff does not limit the waveguide's useful bandwidth. This is not true for $K_d = 9.0$; measurements show that the TE-21 cutoff is lower than that of the TM-01 for filling fractions greater than 0.25.

Fig. 5 is a plot of the TM-01 cutoff frequency divided by that of the TE-11 vs filling fraction for several lining materials. With $k_d = 3.8$ (fused silica) the maximum mode separation is obtained if the filling fraction is about 0.44. Then the second mode's cutoff frequency is 40 per cent above that of dominant mode. By comparison, a homogeneously filled waveguide has a TM-01 cutoff frequency only 31 per cent above the TE-11. For $k_d = 2.1$ (teflon) the optimum filling fraction is about 0.55; this places the TM-01 cutoff 39 per cent above dominant mode cutoff. Fig. 5 also shows some points obtained with $k_d = 9.0$ (alumina). Only a thin lining is of interest here, since a filling fraction greater than about 0.25 brings the TE-21 cutoff below the TM-01.

The dominant mode's equivalent dielectric constant k_{e1} may be calculated from the lowest cavity resonance. The variation of k_{e1} with filling fraction is given in Fig. 6. The region of greatest mode separation is indicated (*).

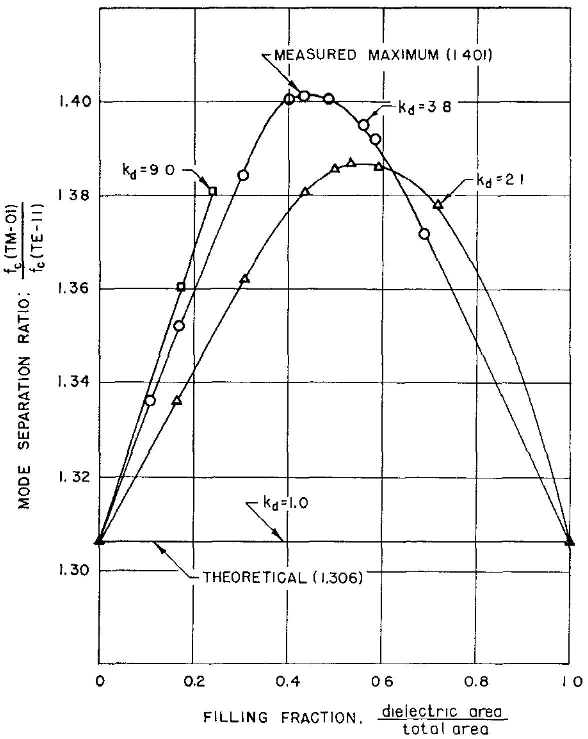


Fig. 5—The relation between mode separation ratio and filling fraction for several lining materials.

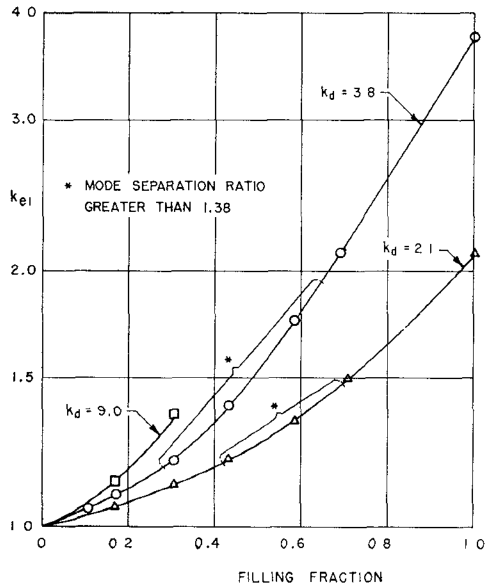


Fig. 6—Equivalent dielectric constant for the dominant mode vs filling fraction.

IV. Guide-Wavelength Variation with Frequency

Assumptions made in Section II have enabled us to specify the frequency behavior of lined waveguide, based on measurements made at discrete frequencies. Let us now examine the validity of these assumptions.

In Section II we assumed that, within some limits, a lined conducting pipe is substantially equivalent to a homogeneously filled conducting pipe of the same diameter and some value of dielectric constant. Once the dielectric constant of the equivalent homogeneous filling has been determined, (1) may be applied to predict the guide wavelength at any frequency. This theory has been investigated by measuring the guide wavelength variation over a wide range of frequencies.

Fig. 7 is a photograph of equipment used to measure guide wavelength. The setup essentially consists of a reflectionless transition from lined circular waveguide to a standard slotted line in rectangular waveguide. A matched termination, shown at the extreme right in the figure, facilitated the design of the transition. A short circuit connected to the circular end of the transition causes a voltage minimum to occur at a certain location in the slotted line. A short length of lined waveguide may now be inserted between the end of the transition and the short circuit. The shift in voltage minimum at the slotted line is a measure of that portion of a guide wavelength contained in the additional length of lined waveguide. Thus the wavelength in the lined waveguide may be measured at various frequencies.

If we can show that the guide wavelength variation agrees with (1), we will have confirmed our hypothesis. Let us rewrite (1) in the form

$$\left(\frac{\lambda_{0c}}{\lambda_g}\right)^2 = k_e\left(\frac{\lambda_{0c}}{\lambda}\right)^2 - 1. \tag{5}$$

Now if we plot our data in the form of $(\lambda_{0c}/\lambda_g)^2$ vs $(\lambda_{0c}/\lambda)^2$, according to our hypothesis, we should obtain a straight line whose slope is k_e and whose ordinate intercept is -1.

Measurements of guide wavelength were performed for a lined waveguide with $k_d = 3.8$ and a filling fraction of 0.64. The observations, plotted as discussed above, are shown in Fig. 8. A dotted straight line is drawn through the lower points, with an ordinate intercept of -1. The slope of this line is 1.8, which agrees with k_e determined by a cavity measurement giving one point in this vicinity. The higher points form a curve departing from this straight line. As the frequency is raised outside the useful band, the discrepancy between measurements, and our simple theory increases.

In Fig. 8, the departure from a straight line has some significance relative to the mode behavior of a lined circular waveguide. It is deduced that the lowest mode, while similar to the simple TE-11 in the frequency range just above its lowest cutoff, has a difference of behavior at higher frequencies, which becomes substantial above the "useful band" of this study. This phenomenon is relevant to the present study only incidentally in that the behavior in the "useful band" departs slightly from the simple waveguide formula. Under the test conditions, this departure introduced a change of less than 6 per cent in guide wavelength.

V. Conclusions

In circular waveguide, the ratio of the TM-01 cutoff frequency over that of the TE-11 dominant mode may be significantly increased by inserting a dielectric lining.

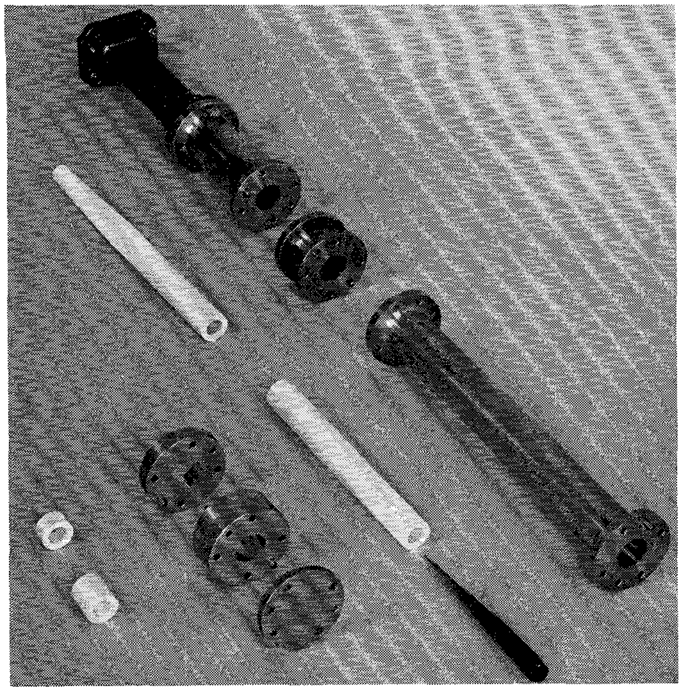


Fig. 7—Photographs of the circular-waveguide test setup.

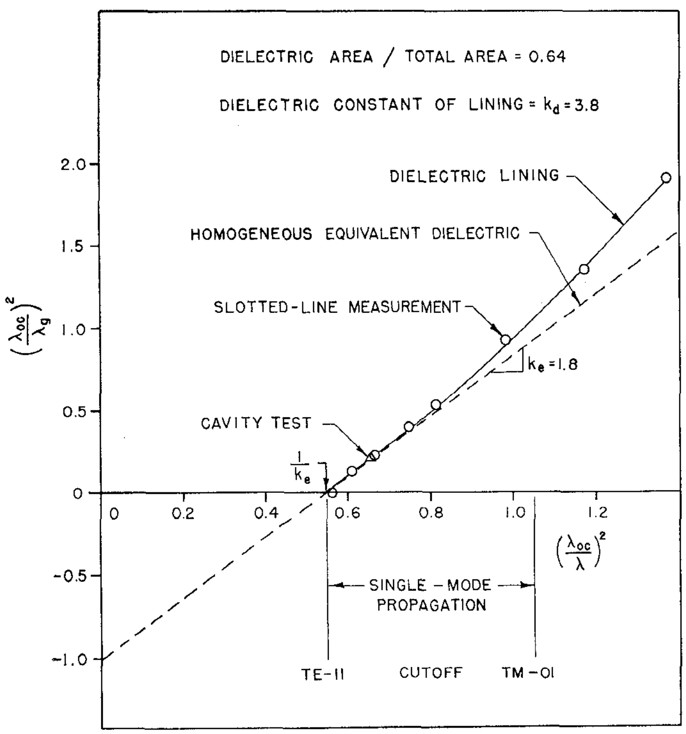


Fig. 8—Graphical determination of equivalent dielectric constant.

If the dielectric constant is 3.8, greatest mode separation is obtained with a dielectric filling fraction about 0.44; then the ratio of the cutoff frequencies may be increased from 1.31 to 1.40. This may enable a twofold increase in the useful bandwidth, since it is preferable to use waveguide above 1.2 times its cutoff frequency. A lining with a dielectric constant in the vicinity of 4 appears to be near the optimum for increasing the single-mode bandwidth. A lining with a value much less than 4 gives less advantage over empty waveguide, while a lining with a value much greater than 4 reduces the TE-21 cutoff to the point where it imposes the principal limitation on the useful bandwidth.

Over the useful bandwidth of lined waveguide, the guide wavelength may be approximately calculated by the simple formula in terms of the equivalent dielectric constant, determined by a cavity measurement.

Applications which would benefit from increased dominant-mode bandwidth in circular waveguide include rotary joints carrying circular polarization, rotary attenuators and phase shifters, and array radiators.

VI. Acknowledgment

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